EARTH SURFACE PROCESSES AND LANDFORMS
Earth Surf. Process. Landforms (2016)

Published online in Wiley Online Library
(wileyonlinelibrary.com) DOI: 10.1002/esp.4004

Uncertainty and sensitivity in a bank stability model: implications for estimating phosphorus loading

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Received 21 October 2015; Revised 19 April 2016; Accepted 7 July 2016

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ESPL
Earth Surface Processes and Landforms

ABSTRACT: Eutrophication of aquatic ecosystems is one of the most pressing water quality concerns in the United States and around the world. Bank erosion has been largely overlooked as a source of nutrient loading, despite field studies demonstrating that this source can account for the majority of the total phosphorus load in a watershed. Substantial effort has been made to develop mechanistic models to predict bank erosion and instability in stream systems; however, these models do not account for inherent natural variability in input values. To quantify the impacts of this omission, uncertainty and sensitivity analyses were performed on the Bank Stability and Toe Erosion Model (BSTEM), a mechanistic model developed by the US Department of Agriculture – Agricultural Research Service (USDA-ARS) that simulates both mass wasting and fluvial erosion of streambanks. Generally, bank height, soil cohesion, and plant species were found to be most influential in determining stability of clay (cohesive) banks. In addition to these three inputs, groundwater elevation, stream stage, and bank angle were also identified as important in sand (non-cohesive) banks. Slope and bank height are the dominant variables in fluvial erosion modeling, while erodibility and critical shear stress had low sensitivity indices; however, these indices do not reflect the importance of critical shear stress in determining the timing of erosion events. These results identify important variables that should be the focus of data collection efforts while also indicating which less influential variables may be set to assumed values. In addition, a probabilistic Monte-Carlo modeling approach was applied to data from a watershed-scale sediment and phosphorus loading study on the Missisquoi River, Vermont to quantify uncertainty associated with these published results. While our estimates aligned well with previous deterministic modeling results, the uncertainty associated with these predictions suggests that they should be considered order of magnitude estimates only. Copyright © 2016 John Wiley & Sons, Ltd.

KEYWORDS: BSTEM; bank erosion; phosphorus; sensitivity; uncertainty

Introduction

Eutrophication of aquatic ecosystems caused by excessive nutrient loading adversely impacts water quality, impairs aquatic habitat, limits recreational opportunities, and increases water treatment costs. Non-point sources, including urban and agricultural stormwater runoff, have been recognized as major contributors of nutrient pollution in the United States while erosion of stream channels has been largely overlooked (but see Lake Champlain TMDL; US EPA, 2015). Phosphorus, which along with nitrogen is a limiting nutrient in freshwater ecosystems (Elser *et al.*, 2007), may enter streams directly adsorbed to eroded soil particles. While in general it has been shown that bank erosion can be a significant contributor of phosphorus to streams (Sekely *et al.*, 2002; Kronvang *et al.*, 2012; Langendoen *et al.*, 2012; Miller *et al.*, 2014), quantification and prediction of these processes is more elusive.

Bank erosion modeling

Significant effort has been made to model bank erosion due to both mass wasting (i.e. geotechnical failure and collapse of banks) and fluvial erosion (i.e. direct entrainment of bank material from flowing water). Other processes such as subaerial erosion (wetting/drying or freeze/thaw cycles) and needle ice formation have been identified but are generally considered less effective erosive forces and are typically not incorporated into bank erosion models (Thorne, 1982; Lawler, 1993; Prosser *et al.*, 2000; Couper and Maddock, 2001). First developed for use with hillslopes, slope-stability relationships exist for both cohesive and non-cohesive soils, resulting in the commonly applied Culmann bank stability relationship for planar failures (Taylor, 1948; Thorne, 1982). Similar efforts have been made for fluvial entrainment. While non-cohesive material can be modeled using a comprehensive force balance (Lane, 1955), cohesive material requires a less direct, excess applied shear stress approach (Partheniades, 1965). Since the development of these basic mechanistic modeling approaches, other investigators have attempted to combine the effects of bank failure and fluvial entrainment into a single model (Osman and Thorne, 1988; Simon *et al.*, 2000; Darby *et al.*, 2007; Langendoen and Alonso, 2008; Langendoen and Simon, 2008). In addition to incorporating the interactions between bank failure and fluvial erosion, these modeling efforts also include other complexities, such as the effects of pore-water pressure, confining force of the in-stream flow, and vegetation. Incorporating more complex physical processes requires more intensive model parameterization, increasing the amount of field data collection required by model users. These models are most effective at the small reach scale and cannot be easily and accurately scaled to predict bank instability and erosion for an entire watershed. However, when considering the importance of these processes in assessing the relative significance of phosphorus loading sources in nutrient management, watershed-scale modeling is essential.

Objectives

The objectives of this study were to:

(1) Identify the Bank Stability and Toe Erosion Model (BSTEM) input parameters that most influence model output (sensitivity analysis). Determining the most important inputs will help model users focus data collection on these variables to achieve the highest possible level of accuracy. We performed a global sensitivity analysis on BSTEM to assess overall model sensitivity. The results of this analysis are useful for future model improvements and development of more broadly applicable bank erosion models with fewer data requirements.
(2) Quantify uncertainty associated with BSTEM estimates from a previous study (uncertainty analysis). Deterministic modeling results in a single output value from a given set of inputs. Probabilistic modeling incorporates variability in inputs by assigning distributions of values rather than single numbers. This results in a distribution of output values that incorporates the given input variability. This approach was applied using data from a previous deterministic BSTEM modeling study (Langendoen *et al.*, 2012) to quantify uncertainty associated with their model results. This uncertainty analysis is distinct and independent of the sensitivity analysis performed under Objective 1.

Bank Stability and Toe Erosion Model (BSTEM)

BSTEM is a mechanistic model developed by the US Department of Agriculture – Agricultural Research Service (USDA-ARS) to predict bank erosion from mass failure and fluvial entrainment (Simon *et al.*, 2000, 2011). Others have provided detailed explanations of BSTEM (Midgley *et al.*, 2012; Daly *et al.*, 2015; Lammers, 2015); we will therefore provide only a general overview. BSTEM consists of two submodels, Bank Stability and Toe Erosion. The Bank Stability model predicts erosion from planar bank failure, using a limit equilibrium analysis to calculate a factor of safety, the ratio of resisting to driving forces acting on the bank. Values greater than one indicate stability while values less than one indicate instability. This model incorporates the stabilizing and destabilizing effects of pore-water pressure (positive pressure decreasing stability and negative pressure increasing stability), increased cohesion due to root reinforcement, and the confining pressure of the streamflow. The Toe Erosion model (something of a misnomer as it considers fluvial erosion across the entire bank, not just the bank toe) uses an excess shear stress equation to calculate erosion rates along a bank face. This model also accounts for increased shear stress on the outside of bends and the effective shear stress acting on individual soil grains. BSTEM can also account for the combined effects of fluvial erosion and mass failure. The eroded bank profile from the Toe Erosion model can be exported to the Bank Stability model to then assess likelihood of failure. A new, dynamic version of BSTEM is under development which automates this process over a given flow record. In order to examine the sensitivity of the Bank Stability and Toe Erosion models separately, only the original, static BSTEM (version 5.4) was utilized in this analysis. This does not impact sensitivity analysis results because the dynamic and static models utilize the same fundamental equations. However, the uncertainty analysis results are impacted by examining the static model only because the linkages between the two submodels are not explicitly incorporated.

Uncertainty and sensitivity analyses

The maxim 'all models are wrong, but some are useful' (Box and Draper, 1987) is popularly used as a recognition of model fallibility; however, it is rarely accompanied by a direct assessment of just how wrong a model is. Given the complexity of the systems they are designed to represent, deterministic environmental models are especially prone to being wrong. It is therefore important to understand not only the assumptions and limitations of the model itself, but also how natural variability within the modeled system complicates the results. Uncertainty in model output comes from three primary sources: (1) how well (or not well) the physical processes being modeled are understood and represented, (2) the simplifying assumptions made by the model, and (3) natural variability in the input parameters and measurement error (Loucks and Van Beek, 2005). These issues may create significant uncertainty in the accuracy of the model results, but few models provide a direct quantification of this uncertainty, making it difficult for users to assign confidence bounds to their results. To help correct this error, and to specifically examine uncertainty associated with point (3), we undertook independent uncertainty and sensitivity analyses of BSTEM. This model was chosen for its inclusion of a number of complex factors (i.e. groundwater and vegetation influences), relative ease of use, and strong mechanistic foundation.

Uncertainty analysis is the process of analyzing the distribution of model output obtained by varying model inputs. Sensitivity analysis quantitatively apportions variance in this output distribution to each of the input variables. Sensitivity analysis may be local or global. Local analyses focus on the sensitivity of the model around known 'true' values of model inputs; however, if there is uncertainty in these 'true' values, a global approach should be used. The global approach examines model sensitivity across the entire possible range of model inputs, therefore this approach was used in this study. Our selected sensitivity analysis methods compute total order effects of input variables, accounting for both the main effect of the input in addition to interaction effects with other variables.

Methods

Input data – sensitivity analysis

Input data for the sensitivity analyses were obtained from a variety of sources and were intended to be representative of the natural range of variability. A summary of all input distributions used in the sensitivity analyses are shown in Tables I and II. We used the ProUCL software (version 5.0.00; Maichle *et al.*, 2013) to test if the distribution of each field data set was normal or lognormal (Shapiro–Wilk [S–W] test) or gamma distributed (Kolmogorov–Smirnov test) at the 95% confidence level. Although BSTEM allows for the inclusion of tension cracks in the Bank Stability model, tension cracks are only incorporated under specific conditions and not for every model run. Therefore, tension cracks were excluded from this analysis as these thresholds effects could impact sensitivity results for other parameters. This confounding impact on all sensitivity analysis results was deemed a greater concern than the lost information on the effects of tension cracks on bank erosion modeling.

Sensitivity analysis

Sensitivity analysis results are dependent on the chosen method (Saltelli *et al.*, 2000, 2004). However, if similar levels of variable importance are observed with different methods, this can increase confidence in the results. For BSTEM, two concerns necessitated the use of multiple sensitivity analysis methods. First, the BSTEM input plant species is a non-numeric variable which is not compatible with all sensitivity methods. Secondly, the Toe Erosion model results in highly skewed output distributions which make it difficult to converge on an accurate sensitivity value. This problem can be avoided by utilizing rank- or log-transformed data; however, variance-based methods cannot support this transformation (Borgonovo *et al.*, 2014). For this reason, a density-based method (Plischke *et al.*, 2013) that is compatible with log-transformed data was used for the Toe Erosion model. Since this method is dependent on numeric inputs, it is not well suited for use on the plant species variable. A variance-based method (Saltelli *et al.*, 2010) that does not require numeric inputs was utilized to analyze the Bank Stability model in order to more accurately capture the effects of plant species on model output. The density-based method of Plischke *et al.* (2013) was also applied to the Bank Stability model (excepting species) to help validate these sensitivity results.

Bootstrapping with 2000 replicates was used for both methods to reduce bias and provide confidence bounds for calculated sensitivity indices. The Sobol′ quasi-random sequence (Sobol′, 1976) was utilized for parameter sampling for both the density and variance methods. BSTEM was modified to run iteratively to facilitate the Monte-Carlo modeling approach required for uncertainty and sensitivity analyses.

Sensitivity analyses were applied to the factor of safety output from the Bank Stability model, average shear stress output from the Toe Erosion model, and eroded area outputs from both. Sensitivity indices for the eroded area output of the Bank Stability model were only computed using the density method when bank failure was predicted (i.e. factor of safety <1). Insufficient sample size precluded the use of the variance method for this output.

Only a single sensitivity analysis was performed for the shear stress output because it is independent of soil-specific parameters. Sensitivity analysis for the eroded area model output was completed for a variety of bank materials: resistant, moderate, and erodible cohesive, coarse and fine sand, and gravel, using only model runs where bank erosion was predicted to occur (i.e. $\tau > \tau_c$). Simplified power regression models were also developed (R software package, version 2.15.1; R Core Team, 2015) using the BSTEM output data to provide another line of evidence for assessing variable importance.

Study area – uncertainty analysis

A Monte-Carlo modeling approach was applied to a field data set from the Missisquoi River watershed in northern Vermont (Langendoen *et al.*, 2012) to quantify uncertainty associated with these previously published model results. The Missisquoi River watershed is approximately 2230 km^2 divided between Vermont, USA (83%) and Quebec, Canada (17%). Predominant land use in the watershed is forest (68%) followed by agriculture (21%) and urban (5%). Historic modification to watershed land use and hydrology, along with direct channel modification, has resulted in significant ongoing channel evolution. The Missisquoi River drains to Lake Champlain, a 1200 km^2 freshwater lake with significant water quality concerns driven by excess sediment and nutrients. Observations of bank erosion along the Missisquoi River and several tributaries led to a watershed-scale BSTEM modeling effort to quantify sediment and phosphorus loading from bank erosion to Lake Champlain (Langendoen *et al.*, 2012). This study used a dynamic version of BSTEM to model sediment and phosphorus loading over 30 years of flow record to Lake Champlain from the US portion of the Missisquoi River and several tributaries under baseline conditions and under various mitigation scenarios. Only baseline conditions were considered for this current analysis.

Uncertainty analysis

A total of 27 bank profiles from the main stem of the Missisquoi River and several major tributaries were used to estimate sediment and phosphorus loading in one or more 2-mile long reaches, extrapolating these model results to the watershed scale (Langendoen *et al.*, 2012). We extracted a representative bank height, bank angle, toe length, and toe angle that best approximated each surveyed profile. Langendoen *et al.* (2012) extrapolated flow data from two long-term United States Geological Survey (USGS) gages on the mainstem of the Missisquoi River with records for water years 1980–2010. They also utilized a simple one-dimensional groundwater model, based on the Darcy Equation, to relate groundwater table movement to river stage. We utilized these given data to develop stage and groundwater elevation distributions (as a percentage of bank height) for each cross-section. These parameters, along with other collected data, were used to model bank erosion and phosphorus loading for each representative site using a Monte-Carlo approach. For most inputs, only single data points were available. We assumed that these variables followed a uniform distribution between 75% and 125% of the given value (or minimum and maximum if multiple points were available).

Total bank phosphorus concentration data were available for 15 of the 27 sites (Howe *et al.*, 2011) and were shown to follow a lognormal distribution (S–W test; $p > 0.05$). Because streambank phosphorus content typically shows a large degree of variability both within and between sites (e.g. Bledsoe *et al.*, 2000; McDowell and Sharpley, 2001; Nellesen *et al.*, 2011), the full lognormal distribution was used for each site. To account for frequency of flows, the stage exceedance probability (estimated by the cumulative lognormal probability function fitted to the given stage data) was multiplied by the calculated Toe Erosion output. For the Bank Stability model, the probability of failure (i.e. the percentage of model runs where failure was predicted) was multiplied by all outputs, yielding annual loading rates by simulating the probability of failure in any given year. See Lammers (2015) for a more detailed descriptions of the methods employed.

Table I. Summary of input data distributions and sources for the sensitivity analyses.

	Distribution type	Mean	SD	Log mean	Log SD	Maximum	Minimum	Source(s)/notes
Bank geometry								
Bank height (m)	Gamma[a]	2.47	1.82	—	—	10	0.5	Andrews, 1984; Hey and Thorne, 1987; Hickin and Nanson, 1984; Williams, 1986
Bank angle (deg)	Normal	60	15	—	—	90	45	Assumed
Bank toe length (% of bank height)	Normal	0.25	0.083	—	—	0.9	0.003	Assumed
Bank toe angle (deg)	Normal	60	15	—	—	90	45	Assumed
Channel and flow parameters								
Channel slope (m/m)	Lognormal	0.0046	0.0050	-5.92	1.12	1.10E-02	7.70E-05	Andrews, 1984; Hey and Thorne, 1987; Hickin and Nanson, 1984
Elevation of flow (% of bank height)	Lognormal	0.528	0.208	-0.71	0.37	1	0.018	USGS gage data (Neuse, White, Yellowstone, Elwha R.)
Duration of flow (hours)	Constant	—	—	—	—	100	100	Constant value
Bank material (for each layer and bank toe)								
Friction angle, ϕ (deg)	Normal							Simon *et al.*, 2011
Cohesion (kPa)	Normal							Simon *et al.*, 2011
Saturated unit weight (kN/m^3)	Normal							Simon *et al.*, 2011
Critical shear stress, T_c (Pa)	Lognormal							BSTEM – distributions based on given values
Erodibility (cm^3/N s)	Lognormal			See Table II				Calculated: $k = 0.1\tau_c^{0.5}$ (Hanson and Simon, 2001)
Matric suction angle, ϕ^b (deg)	Uniform	—	—	—	—	20	10	Fredlund and Rahardjo, 1993
Vegetation rooting effects								
Rooting depth (m)	Normal	1	0.5	—	—	2.5	0.3	Assumed
Species	—	—	—	—	—	—	—	Select randomly from 22 BSTEM options
Age (yrs)	Lognormal	10	5	2.19	0.47	38	1.5	Assumed
Assemblage (%)	Normal	50	25	—	—	100	0	Assume single species with varying coverage
Additional Parameters								
Radius of curvature/channel width (m/m)	Lognormal	3	1.8	0.95	0.56	12.6	0.5	Hickin and Nanson, 1975, 1984; Williams, 1986
Manning's *n*	Lognormal	0.035	0.015	-3.44	0.41	0.1	0.025	Assumed; Chow, 1959
Water-table depth (% of bank height)	Lognormal	0.528	0.208	-0.71	0.37	1	0.018	Same as stage but varies independently

[a]Gamma shape and scale parameters for bank height are 1.914 and 1.293, respectively.

Table II. Summary of soil-specific input data distributions for the sensitivity analyses.

	Soil type	Mean	SD	Log mean	Log SD	Maximum	Minimum
Friction angle (deg)	Sand	29	4.9	—	—	40	20
	Loam	23.3	9.8	—	—	46	6.8
	Clay	18.9	11.1	—	—	43	1
Cohesion (kPa)	Sand	0.5	0.74	—	—	1.9	0.01
	Loam	5.3	4.5	—	—	13.5	0.01
	Clay	8.15	6.6	—	—	26	1
Weight (kN/m^3)	Sand	18.5	0.89	—	—	21.2	15.7
	Loam	18.3	1.3	—	—	22.4	14.2
	Clay	17.6	1	—	—	20.8	14.4
Critical shear stress (Pa)	Gravel	11	5.5	2.3	0.47	42.6	2.3
	Coarse sand	0.51	0.255	-0.78	0.47	1.96	0.11
	Fine sand	0.13	0.065	-2.15	0.47	0.50	0.03
	Erodible cohesive	0.1	0.05	-2.41	0.47	0.38	0.02
	Moderate cohesive	5	2.5	1.5	0.47	19.15	1.05
	Resistant cohesive	50	25	3.8	0.47	191.0	10.5

Results

Sensitivity analysis

Both the density- and variance-based methods were used to assess the sensitivity of the Bank Stability model (factor of safety output) for the three soil types (clay, loam, sand). For both methods, clay and loam showed essentially the same relative variable importance, although quantitative sensitivity indices varied slightly (Table III); therefore, only clay results are shown graphically (Figure 1A). For both soil types, the density method indicated bank height and cohesion were most influential in determining factor of safety. The variance method showed similar trends; however, species emerged as the third most important variable. The eroded area sensitivity results closely mirror that of factor of safety (Table IV), except for higher sensitivity indices for bank height and cohesion, accompanied by subsequent reductions in values for the remaining variables. The sensitivity indices from the two methods show considerable divergence. While the relative rank of each variable is consistent, the variance method results in consistently larger values. This is due to a fundamental difference between the two methods. The density method quantifies differences in conditional probability density functions of model output while the variance method actually apportions model variance to various input variables. The variance-based measure therefore has a more physically realistic output – the percentage of total variance in the model that can be attributed to an individual variable. However, the sum of these values can exceed 1.0 (100% of variance) due to the inclusion of interaction effects.

The results for sand banks differ significantly from those for clay or loam (Figure 1B, Tables III and IV). For factor of safety, groundwater, stage, and bank angle are considered significant along with bank height, cohesion, and species. Sensitivity indices for bank height and cohesion are lower than for clay and loam banks. Groundwater is considered of primary or secondary importance, depending on the sensitivity method. The relative differences in sensitivity indices between input variables are also reduced compared to the clay and loam banks, suggesting variables are more similar in importance. Eroded area results for sand identify bank height as the most important variable, followed by groundwater and cohesion. Other important variables identified during the factor of safety analysis (bank angle and stage) are greatly reduced in importance (Table IV).

The sensitivity analysis for shear stress output from the Toe Erosion model indicated channel slope, bank height, roughness (Manning's *n*), radius of curvature, and stage were (in decreasing order) the most important variables (Figure 2A). Even after bias correction, the sensitivity analysis resulted in non-zero indices for critical shear stress and erodibility, despite the fact that

Table III. Summary of total order sensitivity results from the density and variance methods for factor of safety output for the three bank types.

	Clay				Loam				Sand				
	Rank	Density index	Rank	Variance index	Rank	Density index	Rank	Variance index	Rank	Density index	Rank	Variance index	Average rank
Height	1	0.281	1	0.776	1	0.260	1	0.757	2	0.158	1	0.394	1.2
Cohesion	2	0.189	2	0.340	2	0.193	2	0.358	5	0.101	5	0.133	3.0
Species	[a]	[a]	3	0.313	[a]	[a]	3	0.303	[a]	[a]	6	0.108	4.0
Stage	5	0.066	7	0.064	3	0.096	6	0.067	3	0.123	3	0.240	4.5
Angle	6	0.060	4	0.078	7	0.089	4	0.106	4	0.108	4	0.139	4.8
Friction angle	4	0.069	5	0.067	5	0.094	5	0.102	6	0.062	8	0.053	5.5
Groundwater	7	0.048	12	0.018	6	0.092	11	0.026	1	0.261	2	0.293	6.5
Root depth	3	0.080	14	0.013	4	0.094	13	0.013	7	0.061	7	0.083	8.0
Assemblage	11	0.026	6	0.066	12	0.051	7	0.062	8	0.047	10	0.034	9.0
Toe angle	9	0.032	8	0.024	9	0.056	9	0.034	9	0.047	11	0.015	9.2
Weight	8	0.035	11	0.021	8	0.060	10	0.027	11	0.044	12	0.014	10.0
Toe length	10	0.028	10	0.022	10	0.054	8	0.034	13	0.038	13	0.012	10.7
Age	13	0.023	9	0.022	13	0.050	12	0.025	10	0.046	9	0.041	11.0
ϕ^b	12	0.025	13	0.014	11	0.052	14	0.008	12	0.039	14	0.007	12.7

[a]The density method was not used for the species variable because this is a non-numeric input.

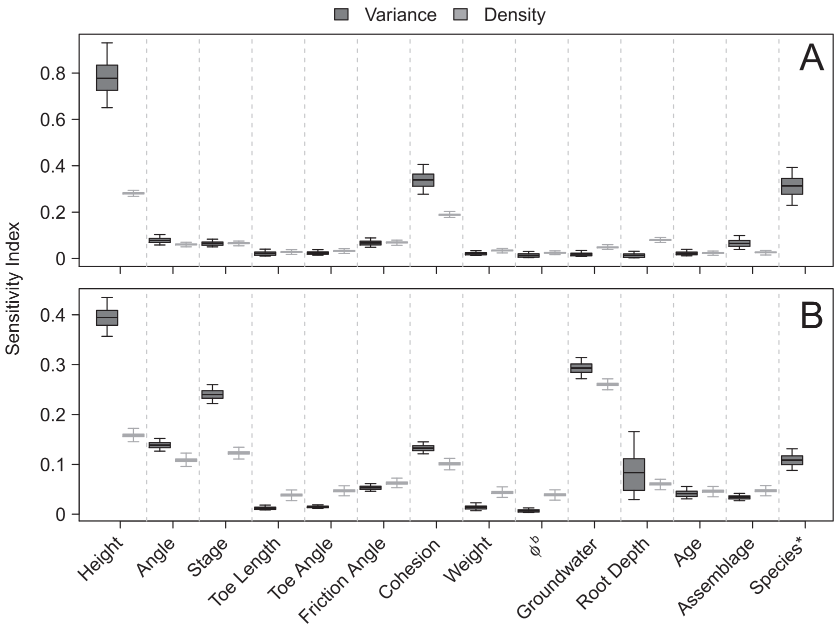

Figure 1. Total order sensitivity indices calculated using both the variance (dark gray) and density (light gray) methods for the factor of safety output for clay banks (A) and sand banks (B), Bank Stability model. Box plots indicate the median, quartiles, and 95% confidence intervals. *The density method was not used for the species variable because this is a non-numeric input.

Table IV. Summary of total order sensitivity results from the density method for the Bank Stability model eroded area output for the three bank types.

	Clay		Loam		Sand		
	Rank	Density index	Rank	Density index	Rank	Density index	Average rank
Height	1	0.521	1	0.476	1	0.380	1.0
Cohesion	2	0.260	2	0.284	3	0.117	2.3
Groundwater	7	0.066	4	0.066	2	0.128	4.3
Friction angle	5	0.077	3	0.070	8	0.032	5.3
Stage	3	0.082	6	0.043	9	0.032	6.0
Angle	4	0.077	8	0.040	7	0.035	6.3
Root depth	12	0.057	5	0.048	4	0.066	7.0
Age	9	0.059	11	0.031	6	0.043	8.7
Assemblage	11	0.057	10	0.037	5	0.046	8.7
Toe angle	6	0.068	9	0.038	11	0.028	8.7
Weight	8	0.064	7	0.040	13	0.024	9.3
ϕ^b	10	0.058	13	0.024	10	0.030	11.0
Toe length	13	0.049	12	0.031	12	0.025	12.3
Species	[a]	[a]	[a]	[a]	[a]	[a]	[a]

[a]The density method was not used for the species variable because this is a non-numeric input.

these variables are not used by BSTEM in the calculation of shear stress. This is likely due to the inability of the bias correction procedure to completely remove the numerical noise associated with the density method. Regardless, the indices for these variables are small (< 0.05) and their similarity to other variables (toe length, toe angle, and bank angle) reinforce the relative unimportance of these remaining inputs.

Generally, similar trends between the bank material types are seen for the eroded area output from the Toe Erosion model, with bank height, slope, stage, radius, and roughness remaining the most important variables in most cases. Toe length, toe angle, and bank angle are the three least important variables in each case (Table V). Due to this similarity, only graphical output data for moderate cohesive material are shown (Figure 2B). Critical shear stress increases in importance when values are higher (resistant cohesive and gravel). For these two bank materials, critical shear stress is ranked higher than erodibility while the opposite is true for the remaining bank types (Table V). However, indices for these variables are still low (< 0.05) indicating that they are still relatively unimportant overall in the model.

Uncertainty analysis

Sediment and phosphorus loading rates were calculated for each reference site using a Monte-Carlo modeling approach and compared to the original results of Langendoen *et al.* (2012) (Figure 3). Extrapolating our model results to the watershed scale gives an estimated mean sediment loading of 23 900 metric tons (t)/yr (interquartile range [IQR]: 11 100–54 700 t/yr) compared to Langendoen *et al.*'s (2012) estimate of 31 600 t/yr. Volumetric loading rates were converted to suspended sediment mass loadings using a median dry density of soil equal to 1285 kg/m^3 and accounting for the percentage of sediment smaller than 125 microns (Langendoen *et al.*, 2012).

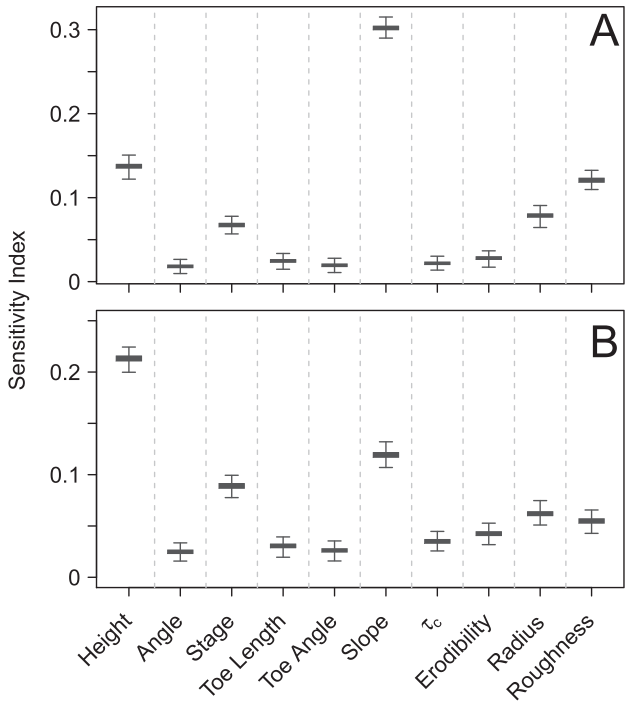

Figure 2. Total order density-based sensitivity indices for shear stress output (A) and eroded area output for moderate cohesive bank soils (B) from the Toe Erosion model. Box plots indicate the median, quartiles, and 95% confidence intervals.

Estimated watershed scale phosphorus loading rates were 41 900 kg/yr (IQR: 19 300–98 100 kg/yr) compared to Langendoen *et al.*'s (2012) estimate of 52 000 kg/yr. The annual average total suspended sediment and total phosphorus load of the Missisquoi River are 88 700 t/yr and 145 000 kg/yr, respectively (1995–2009 estimate; Langendoen *et al.*, 2012). This gives an estimated percent contribution of streambanks to sediment loading of 26% (IQR: 12–59%) compared to the original estimate of 36%. Contributions to phosphorus loading were similar; 29% (IQR: 13–68%) compared to the original estimate of 36%. Despite taking into account fine sediment likely to be transported as suspended load, these percentages of total phosphorus loading attributed to bank erosion may be over-estimates. They are based on total bank phosphorus concentrations, rather than an estimate of water soluble or bioavailable phosphorus which may more accurately reflect in-stream phosphorus concentrations (e.g. Miller *et al.*, 2014). In addition, other uncertain mechanistic processes such as in-stream sediment transport are not incorporated; therefore, our uncertainty bounds may be optimistic as these other sources of uncertainty are not accounted for.

Discussion

Sensitivity analysis – bank stability

Previous studies have performed limited sensitivity analyses on a variety of bank stability models, yielding both similar and dissimilar results. Of the bank material properties, cohesion is considered more important than friction angle or weight (Van de Wiel and Darby, 2007; Parker *et al.*, 2008; Samadi *et al.*, 2009), consistent with the results of this analysis. However, while height was identified as influential by some (Samadi *et al.*, 2009), others determined it was less influential than other parameters such as bank angle and cohesion (Van de Wiel and Darby, 2007). Bank angle was generally recognized as highly significant (Van de Wiel and Darby, 2007; Samadi *et al.*, 2009), contrary to our results which ranked angle no higher than fourth for the factor of safety output. While angle was considered important using high nominal values, its importance has been shown to increase as angle decreases (Van de Wiel and Darby, 2007; Samadi *et al.*, 2009). It is possible that bank angle is more influential in shallow sloped banks and the lower bound of 45° used in this analysis is too large to capture these effects.

The relative importance of groundwater depth also varied among studies, either being considered highly important (Langendoen and Simon, 2008) or nearly insignificant (Samadi *et al.*, 2009). Groundwater was considered more important when cohesion values were low (Langendoen and Simon, 2008), similar to our results for low-cohesive, sandy banks where groundwater was considered the most important variable. It is expected that groundwater increases in importance as cohesion is reduced. In the model, groundwater depth influences the extent and magnitude of matric suction (negative pore-water pressure) within the bank (calculated by BSTEM based on the groundwater table elevation) which influences the apparent cohesion of the bank and subsequent stability. As actual soil cohesion is reduced, other sources of cohesion (including groundwater and vegetation effects) would be expected to increase in importance. Groundwater may be more important for predicting bank erosion rates utilizing a dynamic version of BSTEM, notably by incorporating groundwater table lag with stage changes (Midgley *et al.*, 2012). Furthermore, subsurface flow processes are important for controlling seepage failures (Fox *et al.*, 2007; Fox and Wilson, 2010; Fox and Felice, 2014) and can also reduce the critical shear stress of

Table V. Summary of total order sensitivity results from the density method for the eroded area output of the Toe Erosion model for the six bank types.

	Resistant cohesive		Gravel		Moderate cohesive		Coarse sand		Fine sand		Erodible cohesive		
	Rank	Density index	Rank	Density index	Rank	Density index	Rank	Density index	Rank	Density index	Rank	Density index	Average rank
Height	1	0.178	1	0.201	1	0.213	1	0.267	1	0.293	1	0.303	1
Slope	3	0.066	2	0.104	2	0.119	2	0.152	2	0.151	2	0.159	2.2
Stage	2	0.079	3	0.090	3	0.089	3	0.104	3	0.114	3	0.127	2.8
Radius	4	0.064	4	0.063	4	0.062	5	0.055	5	0.057	5	0.072	4.5
Roughness	6	0.049	5	0.046	5	0.055	4	0.072	4	0.071	4	0.088	4.7
Erodibility	7	0.044	7	0.039	6	0.043	6	0.033	6	0.031	6	0.053	6.3
T_c	5	0.058	6	0.040	7	0.035	7	0.029	7	0.027	7	0.048	6.5
Toe length	10	0.035	9	0.030	8	0.031	8	0.029	8	0.026	8	0.047	8.5
Toe angle	8	0.036	10	0.022	9	0.026	9	0.023	9	0.024	9	0.045	9
Angle	9	0.036	8	0.030	10	0.025	10	0.022	10	0.019	10	0.043	9.5

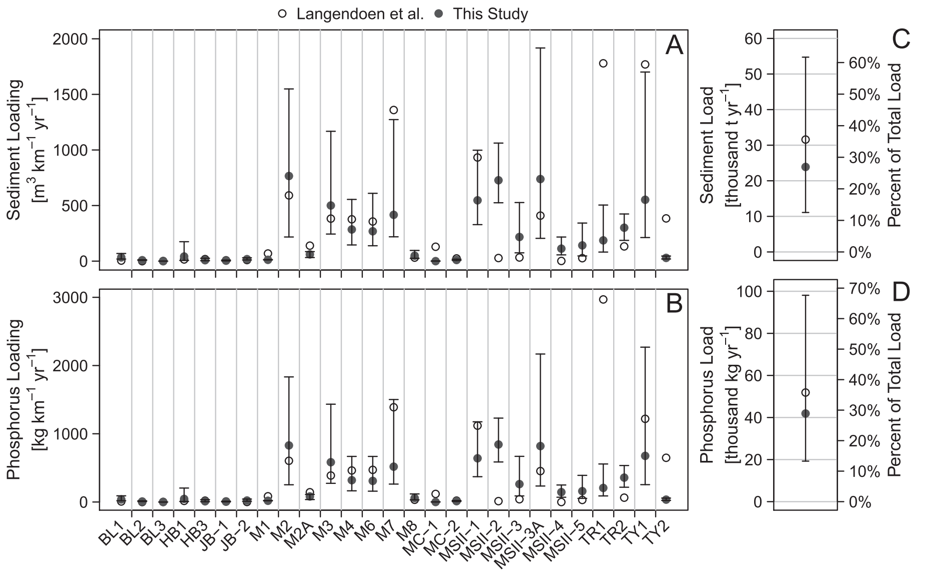

Figure 3. Missisquoi River site-specific sediment (A) and phosphorus (B) loading rates from Langendoen *et al.* (2012) compared to the probabilistic modeling performed in this study. Graphs on the right compare cumulative sediment (C) and phosphorus (D) loading at the watershed scale. Error bars represent the interquartile range of the model output.

the bank material (Chu-Agor *et al.*, 2008; Midgley *et al.*, 2013). These processes are not incorporated within BSTEM and their effects are therefore not reflected in these sensitivity results.

Vegetation has been shown to most influence bank stability at low cohesion and height values (Van de Wiel and Darby, 2007). While vegetation variables (species, root depth, age, and assemblage) were identified as more important for sand banks than clay or loam using the density method, only root depth was ranked higher in sand in the variance method. However, since BSTEM does not allow root depth to exceed bank height, some correlation between these variables was introduced which could artificially increase the sensitivity indices for root depth. Vegetation is generally considered to be a major control on bank stability (Thorne, 1990; Simon and Collison, 2002; Pollen, 2007), leading to the significant effort of incorporating the RipRoot model into BSTEM (Pollen and Simon, 2005). It is therefore surprising that vegetation parameters do not more significantly influence model output. Species type is identified as the third most important variable for clay and loam banks, but only the sixth most important in sand banks.

Examining the output distributions from the density method indicates that for clay banks, two species exhibit the greatest control over model results (data not shown). These species are eastern gammagrass (*Tripsacum dactyloides*) and Alamo switchgrass (*Panacum virgatum*) which have by far the highest maximum root densities of the BSTEM species. A power regression analysis identified these two species as the most influential in predicting factor of safety in clay banks, with coefficients an order of magnitude greater than any other (0.287 and 0.155 compared to 0.009–0.048 for the remaining species). For sand banks, there is far less divergence among species impacting model output, and all species have greater influence overall than for clay banks.

Clearly not all species are created equal, with some having a significant effect on the model while others may be nearly negligible; a relationship that is dependent on specific bank conditions. In addition to root tensile failure (i.e. snapping), BSTEM accounts for root pullout which varies under different soil conditions (e.g. greater frictional resistance to pullout in sand versus clay) which could explain observed differences in relative species importance. Previous BSTEM modeling has shown that woody species, and especially Geyer's willow (*Salix geyeriana*), resulted in the greatest increase in total bank cohesion and that the number of large roots was more important than root density (Polvi *et al.*, 2014). While this study considered only increases in total bank cohesion, both shrubs and trees were shown to result in more stable banks compared to grasses and forbs (Polvi, 2011).

The effects of vegetation on bank stability and erosion are complex and scale dependent. For example, root densities are dependent on soil type and texture, with highly cohesive clays inhibiting rooting while loamy or sandy soils allow for well-developed root networks (Dunaway *et al.*, 1994). This can lead to sandy banks being more stable than clay banks due to variable rooting effects. In addition, significant debate has surrounded the relative stabilizing effects of grasses versus trees (Lyons *et al.*, 2000), a phenomena which is likely scale dependent (Anderson *et al.*, 2004). Trees tend to have higher root densities at depth than grasses (Wynn *et al.*, 2001), a complexity that was not accounted for in this analysis which could explain the relative importance of grass species over tree species in clay banks. Root depth relative to bank height is also an important control on bank stability. Due to higher shallow root densities, grasses may be better at stabilizing short, shallow banks while trees are more effective for tall, steep banks because their roots are able to span the entire bank face (Lyons *et al.*, 2000).

Sensitivity analysis – toe erosion

An unexpected result of the Toe Erosion model sensitivity analysis is the relative unimportance of critical shear stress and erodibility, the two parameters specific to soil type. The basis of the Toe Erosion model is an excess shear stress relationship in which the erosion rate is linearly related to the applied boundary shears stress and the erodibility and critical shear stress of the bank material. This would imply that the Toe Erosion eroded area output has a relatively similar sensitivity to all three of these variables (applied shear stress being a function of bank geometry, channel slope, radius of curvature, and roughness). However, the sensitivity analysis results show that erodibility and critical shear stress are relatively unimportant compared to the input variables that determine applied shear stress. For highly erodible soil types (e.g. fine sand or erodible cohesive), critical shear stress values can be nearly negligible compared to applied shear stress values (as much as three to five orders of magnitude difference). This results in critical shear stress being essentially eliminated from the excess shear stress relationship (Hanson, 1990).

In these instances, applied shear stress becomes by far the dominant term in the excess shear stress equation, explaining the relative importance of variables influencing this value. This also explains why critical shear stress is more influential in soils with higher critical shear stress values (such as resistant cohesive and coarse gravel); the relative difference between critical and applied shear stresses is greatly reduced. Other researchers have also shown that when predicting bank erosion, critical shear stress is increased in importance as grain size (and critical shear stress) increases (McQueen, 2011). Both critical shear stress and erodibility were generally shown to be of low significance in a local sensitivity analysis of BSTEM (Ulary, 2013). However, application of BSTEM to composite streambanks (e.g. a cohesive soil layer overlaying gravel) suggested that accurate prediction of erosion rates were dependent on the erodibility of the cohesive soil layer (Daly *et al.*, 2015) and both the erodibility and the critical shear stress of the underlying gravel layer (Midgley *et al.*, 2012). It should be noted that the sensitivity methods used in this analysis do not account for the important threshold effect of critical shear stress in determining whether erosion occurs (i.e. only when this threshold value is exceeded).

The magnitude of variability of an input variable is directly related to its importance in the model. It is therefore no surprise that slope was consistently identified as an important parameter, given that it varies across three orders of magnitude. Furthermore, specific weight exhibits very little variability (range of ~6 kN/m^3), meaning that even if it significantly influences bank stability, it does not vary enough to considerably impact model output. The low importance of erodibility observed here may also be the result of relatively low variability in this input parameter. However, relative variability in input parameters may be different in a specific study site (e.g. relatively consistent bank heights but wide ranges of cohesion values). Under these conditions, model sensitivity to various inputs may change and the sensitivity results presented here may be less applicable.

Uncertainty analysis

The probabilistic modeling approach acceptably approximated the results of Langendoen *et al.* (2012), considering the simplifications inherent in the analysis. There are some notable exceptions where our modeling approach underestimated the sediment and phosphorus loading rates. This, along with the under-estimate of the cumulative sediment load, may be explained in part by the use of the static rather than the dynamic version of BSTEM. Incorporating feedback between the Toe Erosion and Bank Stability models would likely have led to higher total sediment and phosphorus loading rates as fluvial bank erosion alters bank geometry and potentially increases the incidence of bank failure. In fact, toe erosion has been shown to be a dominant control on bank erosion and instability (Simon *et al.*, 2011). While the linkages between toe erosion and bank stability were not explicitly accounted for in the static model used here, our Monte-Carlo modeling approach did assess bank stability over a range of bank geometries which may crudely simulate alterations from fluvial erosion. In addition, use of a single bank layer can mask the effects of variable soil stratigraphy, although it is difficult to speculate on whether this would result in higher or lower model estimates.

The uncertainty in model outputs varies significantly for each site and generally increases with the median value. The range of variability is also not consistent among sites, with MSII-3 A, M2, M7 and TY1 having significantly larger IQRs compared to other sites. Sites with larger variability in critical shear stress also tend to have more variable outputs, reinforcing the earlier point that when the variability in one input (e.g. critical shear stress) is much larger than other inputs, it may increase in importance. This explains why critical shear stress was identified as a significant source of model uncertainty using the Missisquoi data set but was not in the global sensitivity analysis. Phosphorus trends largely follow those of sediment, unsurprising since the same input distribution of phosphorus concentrations was used for all sites.

Our modeling approach significantly under-predicted both sediment and phosphorus loading for TR1 and TY2 and significantly over-predicted loading for MSII-2. It is difficult to pinpoint the exact source(s) of these inaccuracies but the most likely explanation is the homogeneous bank material assumed in this analysis. These sites had layered banks with significantly different soil-specific parameters. While we attempted to incorporate variability in soil-specific input parameters (e.g. critical shear stress) by weighting the frequency of these values by the thickness of the original bank layer, this does not take into account the spatial orientation of these soils of varying erodibility. A bank consisting entirely of highly erodible material behaves very differently from a bank with erodible material at the top and a less erodible toe. These results indicate that capturing spatial heterogeneity in bank material properties is important for accurately modeling bank erosion. It is well established that cantilever failures (i.e. an undercut toe and overhanging bank) can be a significant failure mechanism (Thorne, 1982) and accurately quantifying differences between various bank layers is important for bank erosion modeling (Midgley *et al.*, 2012; Daly *et al.*, 2015). Further research on the significance of bank layering with different configurations and properties is warranted.

However, the close agreement between model results from other sites demonstrates that if differences in soil properties are less pronounced, the assumption of simplified homogeneous banks still yields accurate results. While examining discrepancies between our results and those of Langendoen *et al.* (2012) is useful, it is not a goal of this analysis to accurately predict sediment and phosphorus loading rates in the Missisquoi River watershed. Rather, we are trying to demonstrate how much uncertainty is inherent in the modeling procedure.

The uncertainty in cumulative annual sediment and phosphorus loads estimates reflect the variable uncertainty observed among modeled sites. For both sediment and phosphorus, the 25th and 75th percentiles are approximately half and twice the median value, respectively. While this is a relatively large error bound, it is lower than might be expected when applying a site-scale model to a relatively large watershed. It may be that order of magnitude estimates (i.e. tens of thousands of metric tons annually) are a more suitable product of this type of analysis than absolute values. Although Langendoen *et al.* (2012) reported a baseline sediment contribution from streambanks of 36%, this analysis indicated the actual value could range from 12% to 59%. Similarly, phosphorus contributions from streambanks were reported as 36% of the total, but could range from 13% to 68%. Others have found that the relative contributions of streambank suspended sediment tend to be around three times the relative contribution of phosphorus (Sekely *et al.*, 2002; Laubel *et al.*, 2003). However, another study from Vermont found similar contributions (31% of total suspended sediment and 25% of total phosphorus) as in the Missisquoi

River watershed (DeWolfe *et al.*, 2004). Including more complexity into the model, notably multi-layered heterogeneous banks and dynamic modeling, would likely have impacted these error ranges. Error bounds may be increased or decreased with the inclusion of bank layering, depending on specific geometry, but would likely increase with dynamic modeling due to feedbacks between the submodels and changes in bank geometry over time.

Implications for managers

The results of this analysis may be of particular interest to managers and model users who wish to use BSTEM to obtain accurate and reliable bank stability and erosion results, but are limited in the amount of field data they can collect due to budget, time, or other constraints. The greatest effort should therefore be made to quantify the most influential variables. Input parameters not only differ in their impact on model output, but also vary in the effort required to accurately measure them. For example, bank height and channel slope are important variables for the Bank Stability (height only) and Toe Erosion models (height and slope). Fortunately, these parameters are relatively easy to measure in the field.

Soil cohesion and critical shear stress, however, are both potentially influential soil-specific parameters that are difficult to measure. Significant effort should be made to accurately quantify these parameters at a variety of points throughout the study area (including in all visible soil layers, if applicable). If modeling stability of sandy or non-cohesive banks, cohesion becomes less important and groundwater more so.

Direct measurement of groundwater is not typically feasible. Therefore, groundwater elevation can either be calculated based on known stages using a simple groundwater model (e.g. Langendoen *et al.*, 2012) or can be assumed to equal stage. This approach ignores the effects of differential stage and groundwater levels on the rising and falling limb of a hydrograph which may impact model accuracy.

Relatively unimportant parameters (i.e. ϕ^b, toe length and angle, soil weight, and non-species vegetation parameters) can be set to nominal or assumed values (or ranges of values) without losing significant explanatory power of the model. Species is the only vegetation parameter to have significant influence on model output; however, this may be largely due to the outsized effect of two species in particular (gamma grass and Alamo switch grass), at least in cohesive banks. Therefore, identifying at least the type of dominant species present (i.e. grass versus tree/shrub) and whether these species are known to have high root densities is important.

Regardless of the difficulty in obtaining field data for specific inputs, spatial and temporal variability is an important consideration. Therefore, every effort should be made to quantify parameters at various points throughout the reach of interest and use these data to pursue a more probabilistic modeling approach. When modeling at a larger scale, subdividing the study area into 'representative reaches' is likely a suitable approach, as long as heterogeneities both within and between sites are accounted for. If a Monte-Carlo approach is not feasible, model users should at least vary inputs at discrete values across the observed range in order to quantify uncertainty associated with their model estimates. If multiple data points are not available for a particular input, this may be varied around the single available value by some fixed uncertainty percentage.

To fully characterize watershed-scale, water-quality impacts, it is important to quantify nutrient loading from all potential sources, including bank erosion. We have demonstrated that there is substantial uncertainty in modeling sediment and nutrient loading from bank erosion, but what is more crucial is how this compares with uncertainty in estimated loads from other sources (e.g. wastewater treatment plants or agricultural runoff). It is essential to compare uncertainty between various loading estimates to adequately assess the relative importance of various nutrient sources in a watershed.

Future research

In this study, we developed a simplified probabilistic version of BSTEM to quantify uncertainty associated with bank stability and erosion modeling. This approach accounts for uncertainty in input values and can provide more robust bank stability analysis than a deterministic modeling approach by providing a probability of failure rather than a single factor of safety value (Parker *et al.*, 2008). We recommend that a complete probabilistic version of BSTEM be developed. Ideally, this would include both the static version utilized herein and the dynamic version currently under development by USDA-ARS.

Even a probabilistic BSTEM is still a site-specific model. To more accurately quantify the potential for bank erosion to contribute to phosphorus loading at the watershed scale, a true watershed-scale model must be developed. Uncertainties and spatial heterogeneity in input values only increase at larger scales and accounting for the effect of this uncertainty on model predictions is critical. These complexities may be addressed in part by watershed stratification into relatively similar modeled reaches and accounting for uncertain input parameters to address inherent heterogeneity.

The results of this analysis can inform development of a simplified watershed-scale model. Specifically, the most influential variables from the Bank Stability model (height, cohesion, groundwater, stage, and species) and Toe Erosion model (slope, height, stage, and in some cases critical shear stress) should be incorporated while other variables may be ignored or set to nominal values. Furthermore, incorporating two bank layers would likely increase model accuracy while adding minimal complexity. Finally, a dynamic model with feedbacks between fluvial erosion and bank stability calculations will provide the most physically meaningful results, especially where banks contain layers with distinct physical properties.

Conclusions

Geomorphic systems are inherently complex and predicting their behavior can be challenging at best. Understanding how this natural complexity manifests itself within a model framework is essential for determining the uncertainty associated with these predictions. We performed a sensitivity analysis of a bank stability and erosion model (BSTEM) to quantify the effects of input parameter uncertainty on model output (Objective 1). We determined that variable importance fluctuates under different conditions (e.g. bank soil type) but identified some general trends. Bank height and cohesion were both identified as influential for predicting stability in banks with more cohesive soils. Groundwater, stage, and bank angle increased in importance as cohesion was reduced. Species type was also considered important, although remaining vegetation parameters tended to have little influence. Parameters used to calculate shear stress (especially slope) were much more influential in modeling fluvial erosion than soil specific parameters; however, critical shear stress is important for predicting the timing of erosion events and its overall impact increases as its range of variability increases relative to other inputs. Applying a probabilistic modeling approach to a previous watershed-scale modeling study allowed for a quantification of uncertainty associated with these published results (Objective 2). While our estimates aligned well with previous model results, the uncertainty associated with these predictions means they should probably be considered order of magnitude estimates only. Given the uncertainty associated with modeling bank erosion, we recommend a probabilistic modeling approach that accounts for the effect of input variability on model output. The results of this study, namely the quantification of variable importance and the impacts of input variability on model output, can also be used to inform the development of a parsimonious watershed-scale model to estimate sediment and nutrient loading from bank erosion.

Acknowledgements—This publication was made possible by the United States Environmental Protection Agency (USEPA) grant 83557001. Its contents are solely the responsibility of the grantee and do not necessarily represent the official views of the USEPA. Further, USEPA does not endorse the purchase of any commercial products or services mentioned in the publication. The authors are grateful to Dan Baker and Ellen Wohl for constructive reviews of an earlier version of this manuscript and three anonymous reviewers whose comments greatly improved the quality of this publication.

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